Math 122 **Name:** Solutions

Joseph C Foster **February 16th, 2018**

Spring 2018 **Time Limit: 50 minutes**

Exam 1

This exam contains 9 pages (including this cover page) and 16 questions.
The total number of marks is 150. You have 50 minutes to complete the exam.

Read each question carefully. When specified, you must show **all** *necessary* work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	5		10	5	
2	5		11	5	
3	5		12	10	
4	5		13	15	
5	5		14	15	
6	5		15	15	
7	5		16	15	
8	5		17	30	
			Total	150	

Total Possible	Your Current Score	Your Current Percentage	Your Current Grade
285			

For questions 1-3, determine if the statement is true or false. By true we mean *always* true.

1. (5 marks) If $f(x) > 0$ then $f'(x) > 0$.

   A. True B. False

2. (5 marks) Linear functions have a constant rate of change.

   A. True B. False

3. (5 marks) $\ln(5a^3) = 3\ln(5a)$

   A. True B. False

For questions 4-7 fill in the blanks.

4. (5 marks) If $C(q)$ represents the cost function, and $R(q)$ represents the revenue function, then the *profit* function is given by

$$\pi(q) = \underline{R(q) - C(q)}.$$

5. (5 marks) If a and k are any numbers such that $z > 0$ and $a \neq 1$, then an exponential function is one of the form

$$f(x) = P_0 a^x.$$

We call P_0 the _____Initial Value_____ and a the _____base or growth factor_____.

6. (5 marks) If an amount P_0 is invested at an annual interest rate r compounded n times a year, then the amount $A(t)$ of the investment after t years is given by the formula

$$A(t) = \underline{P_0\left(1 + r/n\right)^{nt}}.$$

7. (5 marks) The derivative of $f(x)$ at $x = a$ is equal to the _____slope_____ of the _____tangent_____ to $f(x)$ at $x = a$.

For questions 8-10, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 3 marks.

8. (5 marks) When a quantity P changes from P_0 to P_1, we define the *relative change* to be the quantity

A. $\dfrac{P_1 - P_0}{P_0}$

B. $\dfrac{P_0 - P_1}{P_1}$

C. $\dfrac{P_1}{P_0}$

D. $P_1 - P_0$

9. (5 marks) Which of these is *not* a characterisation of the average rate of change of $y = f(x)$ over the interval $[a, b]$.

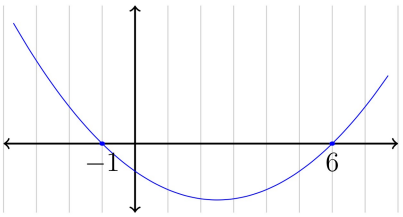
A. Slope of the secant line joining $(a, f(a))$ to $(b, f(b))$

B. $\dfrac{f(b) - f(a)}{b - a}$

C. Slope of the tangent line joining $(a, f(a))$ to $(b, f(b))$

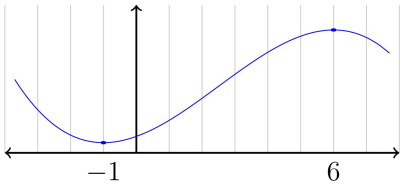
D. $\dfrac{\text{difference in } y}{\text{difference in } x}$

10. (5 marks) The graph of $f'(x)$ is given below.

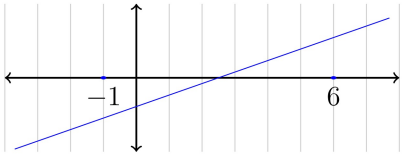


The graph of $f(x)$ *could* look like which of the following?

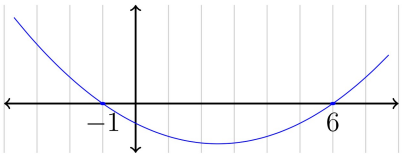
A.



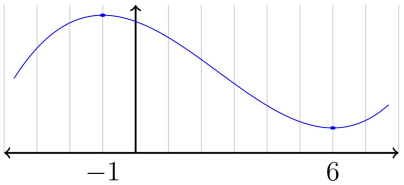
B.



C.



D.



For questions 11-17, show **all** *necessary* work to receive full credit. Please circle or box your final answer. If you cannot complete a problem but can write down what you want to do, and this is correct, you can still receive partial credit. Don't leave anything blank!

11. Kevin invests \$1,293 into a bank account offering 5% per year

(a) (3 marks) Assuming the interest is compounded *annually*, how much will Kevin have in 3 years?

$$1293\left(1+\frac{0.05}{1}\right)^{1\times 3} = \$1496.81$$

(b) (3 marks) Assuming the interest is compounded *monthly*, how much will Kevin have in 3 years?

$$1293\left(1+\frac{0.05}{12}\right)^{12\times 3} = \$1501.78$$

(c) (4 marks) Kevin needs \$2,000 to buy a car. Assuming the interest is compounded *monthly*, in how many *months* will he be able to afford the car?

$$1293\left(1+\frac{0.05}{12}\right)^{12t} = 2000$$

$$12t\ln\left(1+\frac{0.05}{12}\right) = \ln\left(\frac{2000}{1293}\right)$$

$$t = \frac{\ln\left(\frac{2000}{1293}\right)}{12\ln\left(1+\frac{0.05}{12}\right)} \approx 8.74 \text{ years}$$

$$\approx 105 \text{ months}$$

12. Solve each of the following equations using *natural* logarithms

(a) (7 marks) $11 = 5e^{2t}$

$$t = \frac{\ln(11/5)}{2} \approx 0.3942$$

(b) (8 marks) $2e^{3t} = 7e^{5t}$

$$t = \frac{\ln(2/7)}{2} \approx -0.6264$$

13. A function $f(x)$ passes through the points $(1, 5000)$ and $(5, 2048)$. Find an equation for $f(x)$ if

(a) (7 marks) $f(x)$ is a *linear* function.

$$m = \frac{2048 - 5000}{5 - 1} = -738$$

$$f(x) = -738(x-1) + 5000$$

(b) (8 marks) $f(x)$ is an *exponential* function.

$$\frac{2048}{5000} = a^4 = 0.4096$$

$$a = 0.8$$

$$P_0 = \frac{5000}{0.8} = 6250$$

$$f(x) = 6250(0.8)^x$$

14. The population of a city, P, in millions is a function of t, where t is the number of years since 1992. That is, $P = f(t)$.

(a) (5 marks) Explain, in word, what the meaning of $f(10) = 12$ is, in terms of the population of this city.

The population in 2002 was 12 million

(b) (5 marks) The table below gives some of the values of the population of this city.

t	0	5	10	15	20
$f(t)$	3	6	12	24	48

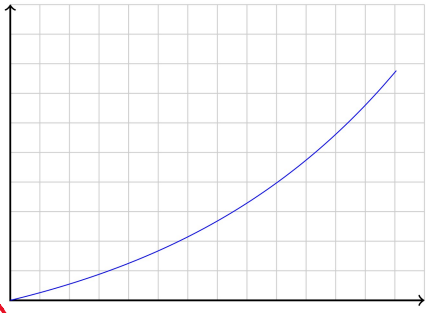
Based on this data, would you say the population is growing linearly or exponentially?

Exp.

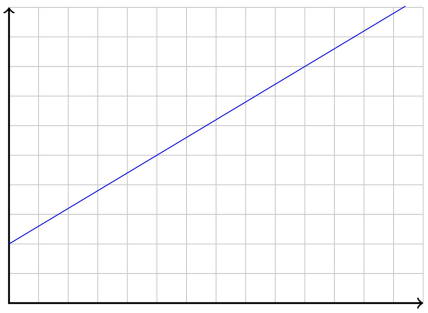
(c) (5 marks) Which of the following graphs do you think best represents this data?

A.

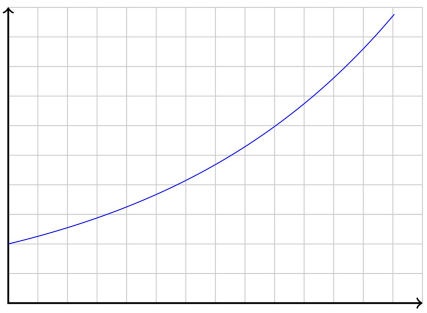
C.



B.



D.



15. Consider the function $f(x) = e^x$.

(a) (2 marks) Find the average rate of change of f between $x = 1$ and $x = 1.1$.

$$\frac{e^{1.1} - e}{0.1} \approx 2.85588$$

(b) (2 marks) Find the average rate of change of f between $x = 1$ and $x = 1.01$.

$$\frac{e^{1.01} - e}{0.01} \approx 2.7319$$

(c) (2 marks) Find the average rate of change of f between $x = 1$ and $x = 1.001$.

$$\frac{e^{1.001} - e}{0.001} \approx 2.7196$$

(d) (2 marks) Find the average rate of change of f between $x = 1$ and $x = 1.0001$.

$$\frac{e^{1.0001} - e}{0.0001} \approx 2.7184$$

(e) (4 marks) Estimate the slope of the tangent line to $f(x)$ at $x = 1$.

$$2.718$$

(f) (3 marks) Estimate $f'(2)$.

$$\frac{e^{2.0001} - e^2}{0.0001} \approx 7.389$$

16. A company manufactures desks. The cost to produce 134 desks is $\$7,102$ and the cost to produce 367 desks is $\$19,451$.

(a) (5 marks) Find a linear cost function $C(q)$ representing the *total* cost of producing q desks.

$$m = \frac{19451 - 7102}{367 - 134} = 53$$

$$C(q) = 53(q - 134) + 7102 = 53q$$

(b) (4 marks) What are the *fixed* costs for this company?

$\$0$

(c) (4 marks) What is the *exact* cost of producing a single desk?

$\$53$

(d) (8 marks) Find the profit function $\pi(q)$ representing the profit made from selling q desks, if the company sells each desk for $\$109$.

$$\pi(q) = R(q) - C(q) = 109q - 53q = 56q$$

This question is continued on the next page.

Continued from previous page.

(e) (5 marks) Find the break-even point for this company.

$q = 0$

(f) (4 marks) The company plans to expand if it can earn over $\$30,000$ in profit. What is the *minimum* number of desks that must be sold to achieve this goal?

$$\frac{30{,}000}{56} = 535.7$$

$\uparrow 536$